### **BUKTI KORESPONDENSI**

### **ARTIKEL JURNAL INTERNASIONAL BEREPUTASI**

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Penulis	: E. Pratidhina, F. Rizky Yuliani, and W. S	: E. Pratidhina, F. Rizky Yuliani, and W. Sunu Brams Dwandaru						
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### Relating Simple Harmonic Motion and Uniform Circular Motion with *Tracker*

Elisabeth Pratidhina<sup>1, 2\*</sup>, Ferina Rizky Yuliani<sup>2</sup>, Wipsar Sunu Brams Dwandaru<sup>1</sup>

<sup>1</sup>Graduate School of Educational Science, Yogyakarta State University

<sup>2</sup>Department of Physics Education, Widya Mandala Catholic University Surabaya

\*Corresponding Email: elisa.founda@gmail.com

### ABSTRACT

In this study, we demonstrate an interesting relationship between simple harmonic motion and uniform circular motion trough a simple experiment. The experiment requires a low cost-easily found materials and free software, *Tracker*. To represent uniform circular motion, we use a tape that is stuck on a fan moving with the constant angular speed. Meanwhile, spring and pendulum motion are used to represent simple harmonic motion. Through Video *Tracker* analysis, we have shown that the positions (*x* and *y* coordinates) of an object undergoes uniform circular motion fit to the sinusoidal function of time, similar to simple harmonic motion. We also analyze the behavior of velocity and acceleration in simple harmonic motion and uniform circular motion. This simple experiment can be used in high school physics courses to lead students in developing a conceptual understanding of uniform circular motion with a less mathematical approach.

# Keyword: simple harmonic motion, uniform circular motion, physics demonstration, *Tracker*

#### **INTRODUCTION**

Recently, there is a rapid development of imaging technology. Moreover, technology in mobile phone has been improved significantly. Nowadays, mobile phones provide various features such as camera, internet, sensor, spreadsheet, game, and so on. The development of imaging technology and camera feature on a mobile phone can be used for physics teaching and learning activity. By

using imaging technology, *Tracker* has become a useful tool in physics education. *Tracker* is free software developed on the Open Source Physics Java code library which can be used for video analysis and physics modeling. *Tracker* includes features such as object tracking with position, velocity, and acceleration overlays and graphs, calibration points and line profiles for analysis of spectra and interference patterns (*"Tracker* Video Analysis and Modeling Tool," 2019).

Some studies have explored the potential of *Tracker* for pedagogical learning tools (Amoroso & Rinaudo, 2018; Poonyawatpornkul & Wattanakasiwich, 2013; Rodrigues, Marques, & Sime, 2016; Wee, Tan, & Leong, 2015). Poonyawatpornkul & Wattanakasiwich (Poonyawatpornkul & Wattanakasiwich, 2013) use *Tracker* to analyze damped harmonic oscillation; Rodrigues (Rodrigues & P, 2014) use *Tracker* to teach optical phenomena; while Loo Kang Wee (Wee et al., 2015) integrate *Tracker* in physics class when discussing free-fall motion. *Tracker* allows students to identify relationships between physical quantities through observation, compare the real world with mathematical modeling, and be trained to construct a model for a particular physical phenomenon (Aguilar-Marin, Chaves-Bacilio, & Jáuregui-Rosas, 2018).

Circular motion and simple harmonic motion are discussed several times in high school physics and introductory college physics courses. Uniform circular motion and simple harmonic motion have close relationships. Simple harmonic motion is a term used when the periodic motion is a sinusoidal function of time. Most students are able to notice simple harmonic motion in the springmass system and the simple pendulum. In fact, the x- and y-coordinates of particle's positions in uniform circular motion are also a sinusoidal function of time, similar to simple harmonic motion. Sometimes students face difficulty to relate uniform circular motion and simple harmonic motion trough geometric and mathematical approaches.

In this work, we show a simple experiment using *Tracker* to compare uniform circular motion and simple harmonic motion. Experiments, representing uniform circular motion and simple harmonic motion, have been conducted with simple materials and tools which can be found easily in daily life. The phenomena were recorded with a mobile phone camera. After that, the videos were analyzed through *Tracker*.

### THEORY

Simple Harmonic Motion in Vertical Spring-Mass System

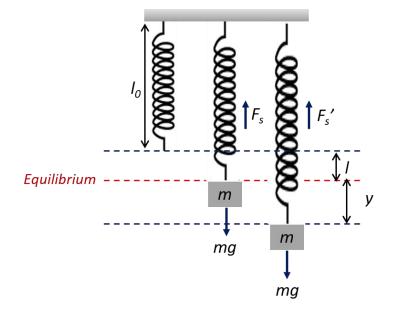


Figure 1. Vertical Spring-Mass System

One of the physical systems representing simple harmonic motion is a vertical spring-mass system. A particle with mass m is attached on vertical spring with an original length of  $l_o$  such as shown in Figure 1. At equilibrium point, the spring force is balanced by the gravitational force, or according to Newton's first law,  $F_s + F_g = 0$ . Thus,

$$-kl + mg = 0$$

Or

$$kl = mg \tag{1}$$

When the spring is stretched down leaving its equilibrium position, there is acceleration, so that  $\Sigma F = ma$ 

$$F_{s}' + F_{g} = ma$$
$$-k(l+y) + mg = m\frac{d^{2}y}{dt^{2}}$$

Since kl = mg,

$$-ky = m\frac{d^2y}{dt^2} \tag{2}$$

Equation (2) can be written as:

$$\frac{d^2y}{dt^2} = -\omega^2 y \tag{3}$$

where,  $\omega^2 = \frac{k}{m}$ .

By solving the second-order differential equation in (3); we yield a solution for the particle's position as follows:

$$y(t) = A\cos(\omega t + \phi) \tag{4}$$

A is called amplitude, whereas  $\phi$  is called initial phase. Both of A and  $\phi$  depend on the initial condition of the particle. As described in equation (4), the particle's periodic motion is a sinusoidal function of time, in which the sinusoidal function is a cosine function. It is the main characteristic of simple harmonic motion. Furthermore, we can determine the velocity and acceleration of the particle by taking the first and second derivative of equation (4), respectively. Thus,

$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi)$$
(5)

$$a(t) = \frac{dv(t)}{dt} = -A\,\omega^2 \cos(\omega t + \phi) \tag{6}$$

### **Uniform Circular Motion**

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t$$

where  $\omega$  is angular speed, whereas  $\phi$  is the initial angular position or initial phase.

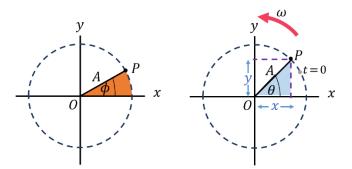


Figure 2. Particle P goes through the uniform circular motion with a constant angular speed of  $\omega$ . The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time t.

Particle *P*, in Figure 2, is moving in a uniform circular motion. It moves in a circle with a radius of *A* and constant angular speed  $\omega$ . The *x*-coordinate of particle P is given by:

$$x(t) = A\cos\theta = A\cos(\omega t + \phi)$$
(7)

The x-coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The x-component of velocity and acceleration are given in equation (8) and (9), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi)$$
(8)

$$a_x(t) = \frac{d}{dt}v_x(t) = -A\omega^2\cos(\omega t + \phi)$$
(9)

The y-coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see equation (10)).

$$y(t) = A\cos\theta = A\sin(\omega t + \phi)$$
(10)

$$v_{y}(t) = \frac{d}{dt}y(t) = -A\omega\cos(\omega t + \phi)$$
(11)

$$a_{y}(t) = \frac{d}{dt}v_{y}(t) = -A\omega^{2}\sin(\omega t + \phi)$$
(12)

### **EXPERIMENTAL SETUP**

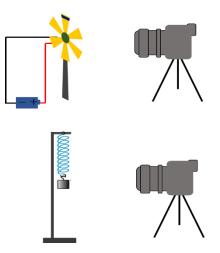


Figure 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.

The experimental setup is simple, as shown in Figure 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Figure 3a). The recorded video is then analyzed with *Tracker* software. Through *Tracker*, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Figure 3b).

#### **RESULT AND DISCUSSION**

The position of an object attached to a fan which moves with constant angular speed has been analyzed through *Tracker* software. As time goes on, the *x*-coordinate of the object goes down and up following cosine function (see Figure 4a) as predicted in theory. Although it is not perfectly smooth, the *x*-component of linear velocity tends to follow sine function with an initial phase ( $\phi$ ) of  $\pi$  radians. Meanwhile, the *x*-component of acceleration tends to follow negative cosine function, consistent with equation (7).

As shown in Figure 5, the *y*-coordinate of the object on uniform circular motion smoothly fits with the sine function in equation (8). On the other hand, the *y*-component of velocity and acceleration graph are not as smooth as the position graph. However, the *y*-component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

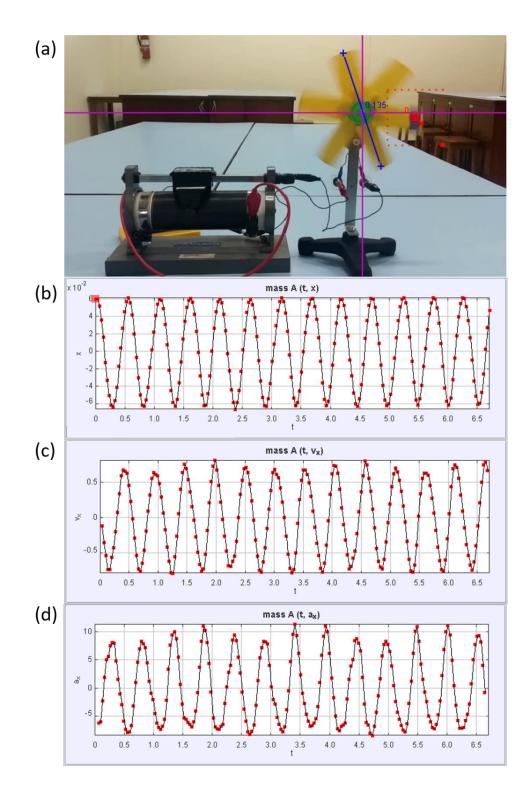


Figure 4. (a) uniform circular motion experiment using an electric fan (b) x-position versus time graph, (d), x-component linear velocity versus time graph, (e) x-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI unit.

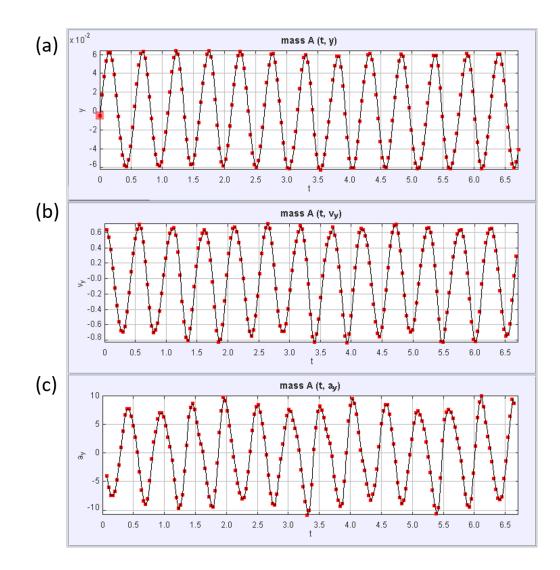


Figure 5. (a) *y*-position versus time graph, (c), *y*-component linear velocity versus time graph, (d) *y*-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI unit.

In addition, we did experiment with a well-known simple harmonic motion system, i.e. massspring system and simple pendulum. Figure 6a-c shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ , whereas the acceleration fits cosine function or sine function with  $\phi = \frac{\pi}{2}$ .

The analysis of simple pendulum motion is presented in Figure 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The ball was swung to the left initially. The position of the ball perfectly follows the sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ . Meanwhile, the acceleration graph fits the sine function with  $\phi = \frac{\pi}{2}$ .

We have shown through analysis with *Tracker* that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

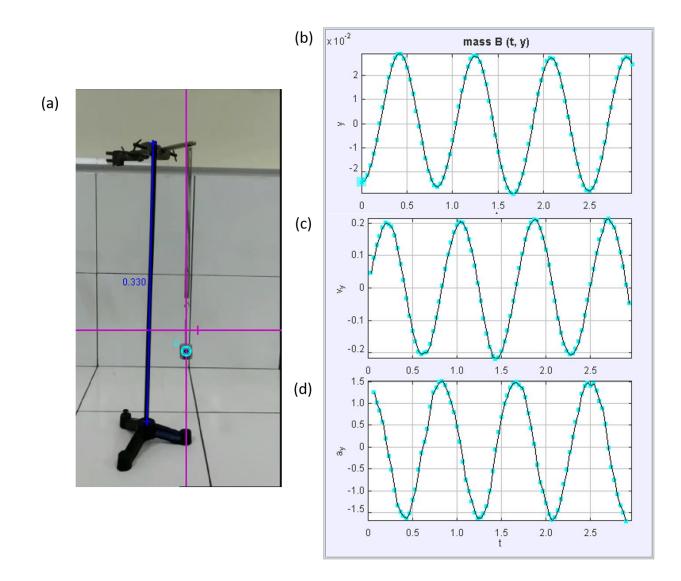


Figure 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) *y*-position, (c) *y*-component velocity, (d) *y*-component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI unit.

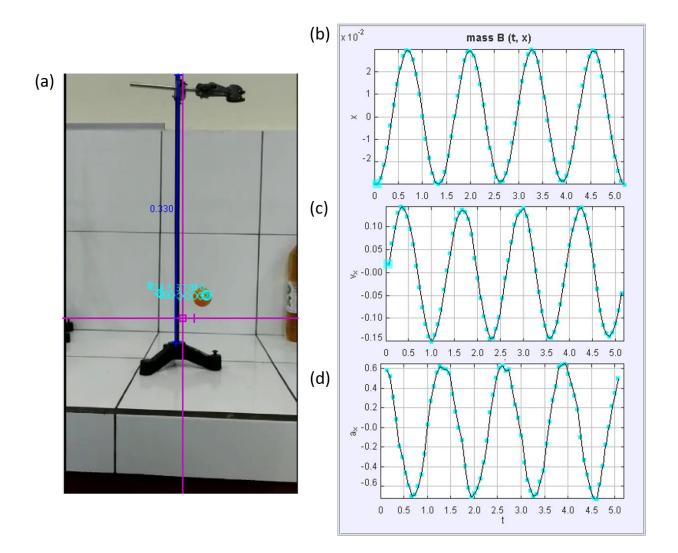


Figure 7. (a) simple pendulum experiment, (b) *x*-position, (c) *x*-component velocity, (d) *x*component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI unit.

### STRATEGIC IMPLEMENTATION IN CLASS

A simple experiment with *Tracker* software is potential to be embedded in a physics classroom to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with *Tracker* software. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as projects, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

### CONCLUSION

In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using *Tracker* with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The x- and y-coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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### 2. Bukti konfirmasi review dan hasil review 18 Februari 2020 (accepted with minor revision)



#### [RMF E] Editor Decision

6 messages

Prof. Alejandro Ayala <rmf@ciencias.unam.mx> To: Ms Elisabeth FOUNDA Pratidhina <elisa.founda@gmail.com>

Dear Ms Elisabeth FOUNDA Pratidhina:

We are pleased to inform you that your paper titled: "Relating Simple Harmonic Motion and Uniform Circular Motion with Tracker", has been accepted for publication in Revista Mexicana de Física E.

Comments from the referee are enclosed for your information. Please notice that the reviewer suggests that for the final version you correct a typo in Fig. 5.

Sincerely,

Prof. Alejandro Ayala Chief Editor Revista Mexicana de Física

Reviewer A:

Subject: Relating Simple Harmonic Motion and Uniform Circular Motion with Tracker

Reviewer A:

Subject: Relating Simple Harmonic Motion and Uniform Circular Motion with Tracker

The manuscript is well written, it only has one typo. The authors properly cite past literature with similar findings to theirs, so comparing their findings to these studies in particular would be most helpful to the reader. The manuscript is concise and easy to understand. Effectively presents a demonstrative interesting relationship between uniform circular motion and simple harmonic motion trough simple experiment using the free software Tracker. I found especially valuable how they have shown that uniform circular motion has symmetry with simple harmonic motion through VideoTracker analysis, where they have shown that the positions of an object undergoes uniform circular motion fit to sinusoidal function of time, as same as, the one shown in a simple harmonic motion. Comparing the analytical and experimental method the results are consistent. Also, are very helpful experiments for students and teachers of engineering faculty. The article could be published with a improvement:

Minor typographical improvement:

In the Figure 5 in the incise (c) should be replaced "compenent" by "component".

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Elisabeth Founda <elisa.founda@gmail.com> 1 To: "Prof. Alejandro Ayala" <rmt@dencias.unam.mx></rmt@dencias.unam.mx></elisa.founda@gmail.com>											B, 2020 (	at 8:48 PM	
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Elisabeth Founda <elisa.founda@gmail.com> To: "Prof. Alejandro Ayala" <rmf@ciencias.unam.mx>

Dear. Prof. Alejandro Ayala

We have read the reviewer comments and revised our manuscript. We make some changes on the sentences and correct the typo. In addition, we also revised the authors' affiliation in the manuscript The revised manuscript is attached in this email

Thank you

[Quoted text hidden] --With sincere regards,

Elisabeth Pratidhina Founda Noviani

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Revista Mexicana de Fisica <rmf@ciencias.unam.mx> to me •

Dear Ms Elisabeth FOUNDA Pratidhina:

We acknowledge receipt the file.

We will now begin the technical editing process that includes layout and style correction. This process is carried out for all received papers in strict chronological order according to the reception date of the aforementioned files. The technical editor will contact and send you the galley proof. This process can take a bit more than a couple of weeks, this means that the status of your work in our platform during this time will be "in editing".

We thank you in advance for your understanding.

Sincerely, Editorial Team Revista Mexicana de Física. Tue, Feb 18, 2020 at 9:56 PM

Sat, Feb 22, 2020, 3:36 AM

### Relating Simple Harmonic Motion and Uniform Circular Motion with *Tracker*

Elisabeth Pratidhina<sup>1, 2\*</sup>, Ferina Rizky Yuliani<sup>2</sup>, Wipsar Sunu Brams Dwandaru<sup>1</sup>

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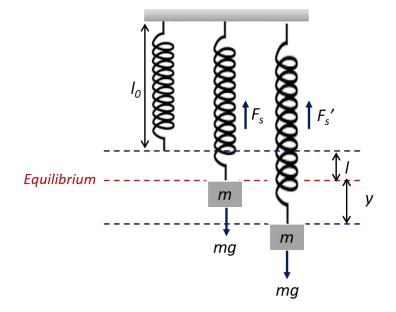


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$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi)$$
(5)

$$a(t) = \frac{dv(t)}{dt} = -A\,\omega^2 \cos(\omega t + \phi) \tag{6}$$

### **Uniform Circular Motion**

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t$$

where  $\omega$  is angular speed, whereas  $\phi$  is the initial angular position or initial phase.

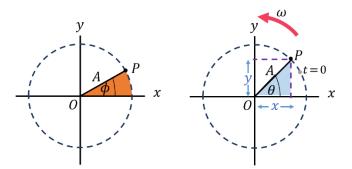


Figure 2. Particle P goes through the uniform circular motion with a constant angular speed of  $\omega$ . The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time t.

Particle *P*, in Figure 2, is moving in a uniform circular motion. It moves in a circle with a radius of *A* and constant angular speed  $\omega$ . The *x*-coordinate of particle P is given by:

$$x(t) = A\cos\theta = A\cos(\omega t + \phi)$$
(7)

The x-coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The x-component of velocity and acceleration are given in equation (8) and (9), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi)$$
(8)

$$a_x(t) = \frac{d}{dt}v_x(t) = -A\omega^2\cos(\omega t + \phi)$$
(9)

The y-coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see equation (10)).

$$y(t) = A\cos\theta = A\sin(\omega t + \phi)$$
(10)

$$v_{y}(t) = \frac{d}{dt}y(t) = -A\omega\cos(\omega t + \phi)$$
(11)

$$a_{y}(t) = \frac{d}{dt}v_{y}(t) = -A\omega^{2}\sin(\omega t + \phi)$$
(12)

### **EXPERIMENTAL SETUP**

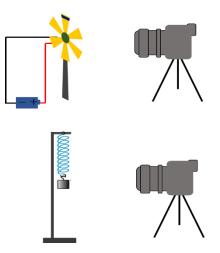


Figure 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.

The experimental setup is simple, as shown in Figure 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Figure 3a). The recorded video is then analyzed with *Tracker* software. Through *Tracker*, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Figure 3b).

#### **RESULT AND DISCUSSION**

The position of an object attached to a fan which moves with constant angular speed has been analyzed through *Tracker* software. As time goes on, the *x*-coordinate of the object goes down and up following cosine function (see Figure 4a) as predicted in theory. Although it is not perfectly smooth, the *x*-component of linear velocity tends to follow sine function with an initial phase ( $\phi$ ) of  $\pi$  radians. Meanwhile, the *x*-component of acceleration tends to follow negative cosine function, consistent with equation (7).

As shown in Figure 5, the *y*-coordinate of the object on uniform circular motion smoothly fits with the sine function in equation (8). On the other hand, the *y*-component of velocity and acceleration graph are not as smooth as the position graph. However, the *y*-component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

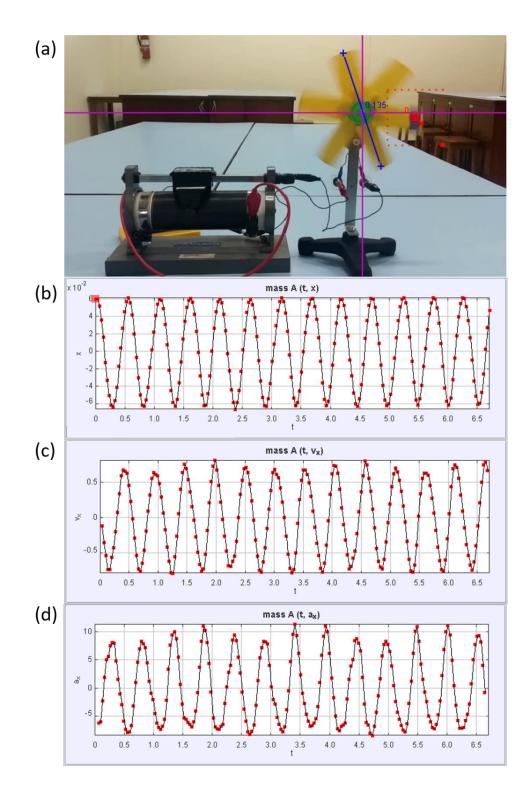


Figure 4. (a) uniform circular motion experiment using an electric fan (b) x-position versus time graph, (d), x-component linear velocity versus time graph, (e) x-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI unit.

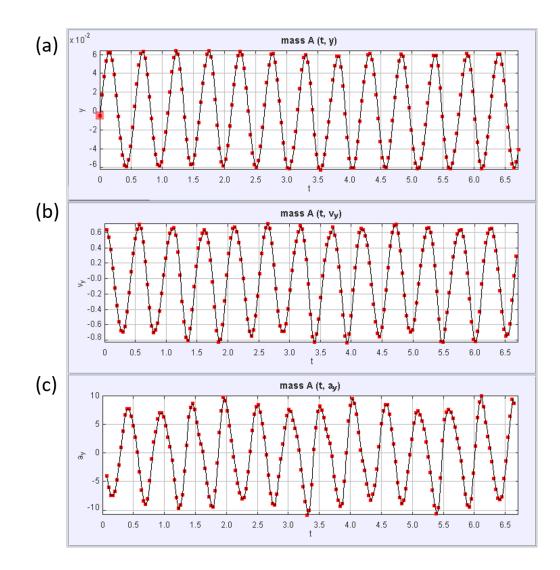


Figure 5. (a) *y*-position versus time graph, (c), *y*-component linear velocity versus time graph, (d) *y*-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI unit.

In addition, we did experiment with a well-known simple harmonic motion system, i.e. massspring system and simple pendulum. Figure 6a-c shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ , whereas the acceleration fits cosine function or sine function with  $\phi = \frac{\pi}{2}$ .

The analysis of simple pendulum motion is presented in Figure 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The ball was swung to the left initially. The position of the ball perfectly follows the sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ . Meanwhile, the acceleration graph fits the sine function with  $\phi = \frac{\pi}{2}$ .

We have shown through analysis with *Tracker* that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

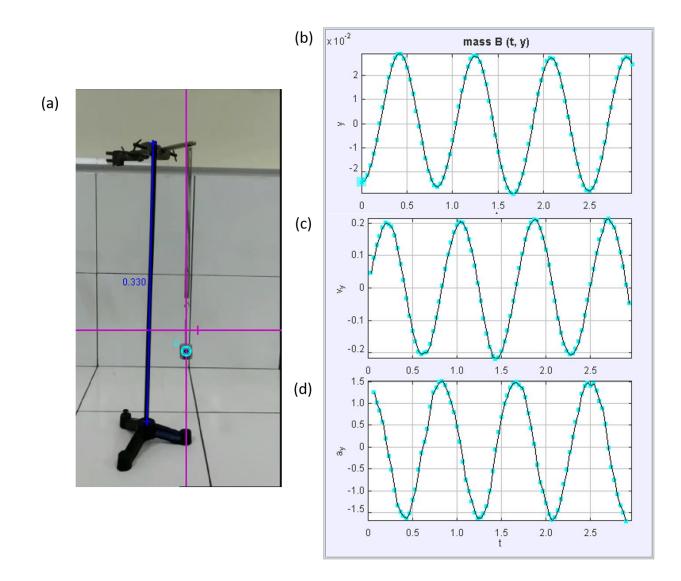


Figure 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) *y*-position, (c) *y*-component velocity, (d) *y*-component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI unit.

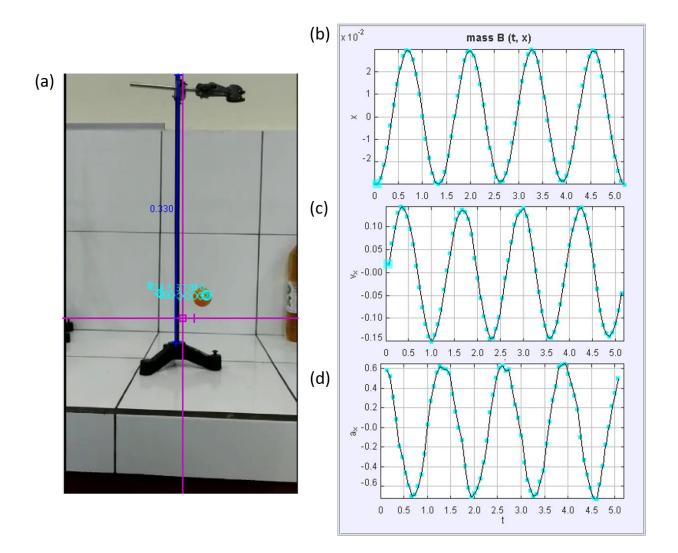


Figure 7. (a) simple pendulum experiment, (b) *x*-position, (c) *x*-component velocity, (d) *x*component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI unit.

### STRATEGIC IMPLEMENTATION IN CLASS

A simple experiment with *Tracker* software is potential to be embedded in a physics classroom to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with *Tracker* software. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as projects, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

### CONCLUSION

In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using *Tracker* with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The x- and y-coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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E. Pratidhina *et al., Relating simple harmonic motion and uniform circular motion with Tracker* 

Q1: between  $\rightarrow$  between the Q2: low cost-easily found materials  $\rightarrow$  low cost, easily-found materials, Q3: software, *Tracker*  $\rightarrow$  software Tracker Q4: Video Tracker analysis  $\rightarrow$  video analysis with Tracker Q5: phone  $\rightarrow$  phones Q6: camera, internet, sensor, spreadsheet, game,  $\rightarrow$  cameras, internet, a variety of sensors, spreadsheets, games, Q7: camera feature on a mobile phone  $\rightarrow$  the inclusion of cameras on mobile phones Q8: activity  $\rightarrow$  activities **Q9**: *Tracker*  $\rightarrow$  Tracker Q10: Tracker is free  $\rightarrow$  Tracker is a free Q11: *Tracker* → Tracker Q12: Tracker → Tracker Q13: Poonyawatpornkul and Wattanakasiwich Q14: Tracker → Tracker Q15: Rodrigues and Simeão Carvalho Q16: Tracker → Tracker Q17: delete "while" Q18: Wee et al. Q19: Tracker → Tracker Q20: class  $\rightarrow$  lessons Q21: Tracker → Tracker Q22: particle's positions  $\rightarrow$  particles' positions Q23: Tracker → Tracker Q24: Tracker → Tracker Q25: m → \$m\$ Q26: by  $\rightarrow$  with Q27: insert comma after \$ma\$ Q28: include equation number Q29: replace period with comma at the end of equation Q30: remove indent Q31: remove comma Q32: (3);  $\rightarrow$  Eq. (3), Q33: insert comma at the end of equation Q34: A in called amplitude, whereas  $\phi$  is called initial phase.  $\rightarrow$  where A is the amplitude and \$\phi\$ is the initial phase. Q35: (acceleration should be the second derivative of the position or the first derivative of the velocity) Q36: remove comma Q37: remove comma

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Q39: Eq.  $\rightarrow$  Eqs. Q40: replace period with comma and insert comma and period after Eqs. (9) and (10), respectively Q41: replace period with comma Q42: Tracker software → Tracker Q43: Tracker → Tracker Q44: unit  $\rightarrow$  units Q45: unit  $\rightarrow$  units Q46: Tracker software → Tracker Q47:  $y \rightarrow \$y\$$ Q48: did  $\rightarrow$  performed an Q49: *i.e.*  $\rightarrow$  i.e., Q50: unit  $\rightarrow$  units Q51: unit  $\rightarrow$  units Q52: Tracker → Tracker Q53: Tracker software is potential to be embedded in a physics classroom  $\rightarrow$  Tracker could potentially be included in physics classrooms Q54: Tracker software → Tracker Q55: Tracker → Tracker

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Q5

#### Relating simple harmonic motion and uniform circular motion with Tracker

E. Pratidhina<sup>a,b,\*</sup>, F. Rizky Yuliani<sup>b</sup>, and W. Sunu Brams Dwandaru<sup>a</sup> <sup>a</sup>Graduate School of Educational Science, Yogyakarta State University. <sup>b</sup>Department of Physics Education, Widya Mandala Catholic University Surabaya. \*e-mail: elisa.founda@gmail.com

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In this study, we demonstrate an interesting relationship between simple harmonic motion and uniform circular motion trough a simple experiment. The experiment requires a low cost-easily found materials and free software, Tracker. To represent uniform circular motion, we use a tape that is stuck on a fan moving with the constant angular speed. Meanwhile, spring and pendulum motion are used to represent simple harmonic motion. Through Video Tracker analysis, we have shown that the positions (x and y coordinates) of an object undergoes uniform circular motion fit to the sinusoidal function of time, similar to simple harmonic motion. We also analyze the behavior of velocity and acceleration in simple harmonic motion and uniform circular motion. This simple experiment can be used in high school physics courses to lead students in developing a conceptual understanding of uniform circular motion with a less mathematical approach.

Keywords: Simple harmonic motion; uniform circular motion; physics demonstration; Tracker

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#### 1. Introduction

Recently, there is a rapid development of imaging technology. Moreover, technology in mobile phone has been improved significantly. Nowadays, mobile phones provide various features such as camera, internet, sensor, spreadsheet, game, and so on. The development of imaging technology and camera feature on a mobile phone can be used for physics teaching and learning activity. By using imaging technology, Tracker has become a useful tool in physics educa-Q10 tion. Tracker is free software developed on the Open Source Physics Java code library which can be used for video analysis and physics modeling. Tracker includes features such as object tracking with position, velocity, and acceleration overlays and graphs, calibration points and line profiles for analysis of spectra and interference patterns [6].

Some studies have explored the potential of Tracker for pedagogical learning tools [2-4,7]. Poonyawatpornkul Wattanakasiwich [3] use Tracker to analyze damped har-Q15 monic oscillation; Rodrigues [5] use Tracker to teach optical phenomena; while Loo Kang Wee [7] integrate Tracker O20in physics class when discussing free-fall motion. Tracker allows students to identify relationships between physical quantities through observation, compare the real world with mathematical modeling, and be trained to construct a model for a particular physical phenomenon [1].

> Circular motion and simple harmonic motion are discussed several times in high school physics and introductory college physics courses. Uniform circular motion and simple harmonic motion have close relationships. Simple harmonic motion is a term used when the periodic motion is a sinusoidal function of time. Most students are able to notice simple harmonic motion in the spring-mass system and the simple pendulum. In fact, the x- and y-coordinates of particle's positions in uniform circular motion are also a sinusoidal function of time, similar to simple harmonic motion.

Sometimes students face difficulty to relate uniform circular motion and simple harmonic motion trough geometric and mathematical approaches.

In this work, we show a simple experiment using Tracker to compare uniform circular motion and simple harmonic motion. Experiments, representing uniform circular motion and simple harmonic motion, have been conducted with simple materials and tools which can be found easily in daily life. The phenomena were recorded with a mobile phone camera. After that, the videos were analyzed through Tracker.

#### 2. Theory

or

#### 2.1. Simple harmonic motion in vertical spring-mass system

One of the physical systems representing simple harmonic motion is a vertical spring-mass system. A particle with mass **m** is attached on vertical spring with an original length of  $l_o$  Q25 such as shown in Fig. 1. At equilibrium point, the spring force is balanced by the gravitational force, or according to Newton's first law,  $F_s + F_q = 0$ . Thus,

-kl + mg = 0,

(1)

When the spring is stretched down leaving its equilibrium position, there is acceleration, so that  $\sum F = ma$ 

kl = mq.

 $\boldsymbol{F}_s + \boldsymbol{F}_q = \boldsymbol{m}a,$  $-k(l+y) + mg = m\frac{d^2y}{dt^2}$ 

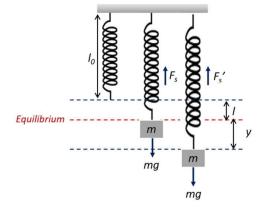


FIGURE 1. Vertical Spring-Mass System.

Since kl = mg,

$$-ky = m\frac{d^2y}{dt^2}.$$

230 Equation (2) can be written as:

$$\frac{d^2y}{dt^2} = \omega^2 y,\tag{3}$$

where,  $\omega^2 = k/m$ .

By solving the second-order differential equation in (3); we yield a solution for the particle's position as follows:

$$y(t) = A\cos(\omega t + \phi) \tag{4}$$

A is called amplitude, whereas  $\phi$  is called initial phase. Both of A and  $\phi$  depend on the initial condition of the particle. As described in Eq. (4), the particle's periodic motion is a sinusoidal function of time, in which the sinusoidal function is a cosine function. It is the main characteristic of simple harmonic motion. Furthermore, we can determine the velocity and acceleration of the particle by taking the first and second derivative of Eq. (4), respectively. Thus,

$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi),$$

$$a(t) = \frac{dy(t)}{dt} = -A\omega^2\sin(\omega t + \phi).$$
(6)

2

#### 2.2. Uniform circular motion

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t,\tag{7}$$

where  $\omega$  is angular speed, whereas  $\phi$  is the initial angular position or initial phase.

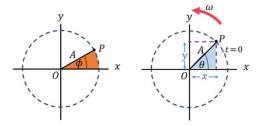


FIGURE 2. Particle P goes through the uniform circular motion with a constant angular speed of  $\omega$ . The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time t.

Particle P, in Fig. 2, is moving in a uniform circular motion. It moves in a circle with a radius of A and constant angular speed  $\omega$ . The x-coordinate of particle P is given by:

$$x(t) = A\cos\theta = A\cos(\omega t + \phi).$$
(8)

Q40

The x-coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The x-component of velocity and acceleration are given in Eq. (8) and (9), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega t\sin(\omega t + \phi)$$
(9)

$$a_x(t) = \frac{d}{dt}v_x(t) = -A\omega^2 t\cos(\omega t + \phi)$$
(10)

The y-coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see Eq. (10)).

$$y(t) = A\cos\theta = A\sin(\omega t + \phi), \tag{11}$$

$$v_x(t) = \frac{d}{dt}y(t) = -A\omega\cos(\omega t + \phi), \qquad (12)$$

$$a_y(t) = \frac{d}{dt}v_y(t) = -A\omega^2 t\sin(\omega t + \phi).$$
(13)

### 3. Experimental setup

The experimental setup is simple, as shown in Fig. 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Fig. 3a). The recorded video is then analyzed with Tracker software. Through Tracker, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Fig. 3b).

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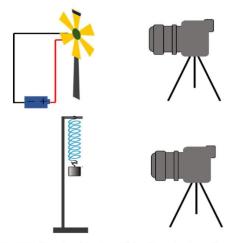
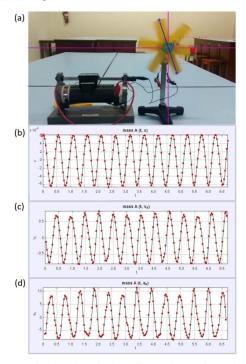
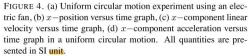


FIGURE 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.





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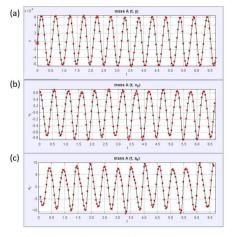


FIGURE 5. (a) y-position versus time graph, (b) y-component linear velocity versus time graph, (c) y-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI unit.

### 4. Result and discussion

The position of an object attached to a fan which moves with constant angular speed has been analyzed through *Tracker* software. As time goes on, the x-coordinate of the object goes down and up following cosine function (see Fig. 4a) as predicted in theory. Although it is not perfectly smooth, the x-component of linear velocity tends to follow sine function with an initial phase ( $\phi$ ) of  $\pi$  radians. Meanwhile, the x-component of acceleration tends to follow negative cosine function, consistent with Eq. (7).

As shown in Fig. 5, the y-coordinate of the object on uniform circular motion smoothly fits with the sine function in Eq. (8). On the other hand, the y-component of velocity and acceleration graph are not as smooth as the position graph. However, the **y**-component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

In addition, we did experiment with a well-known simple harmonic motion system, *ice* mass-spring system and simple pendulum. Figure 6a-c) shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with  $\phi = -\pi$  radians. The velocity fits cosine function with  $\phi = -\pi/2$ , whereas the acceleration fits cosine function or sine function with  $\phi = \pi/2$ .

The analysis of simple pendulum motion is presented in Fig. 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The

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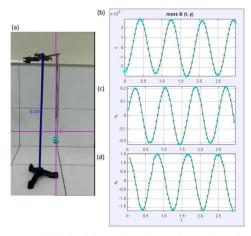


FIGURE 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) y—position, (c) y—component velocity, (d) y—component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI unit.

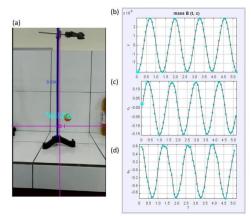


FIGURE 7. (a) simple pendulum experiment, (b) x-position, (c) x-component velocity, (d) x-component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI unit.

ball was swung to the left initially. The position of the ball perfectly follows the sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\pi/2$ . Meanwhile, the

acceleration graph fits the sine function with  $\phi = \pi/2$ .

We have shown through analysis with *Tracker* that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

#### 5. Strategic implementation in class

A simple experiment with *Tracker* software is potential to be embedded in a physics classroom to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with *Tracker* software. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as projects, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

#### 6. Conclusion

In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using *Tracker* with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The x- and y-coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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# Q56-62

RELATING SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION WITH TRACKER

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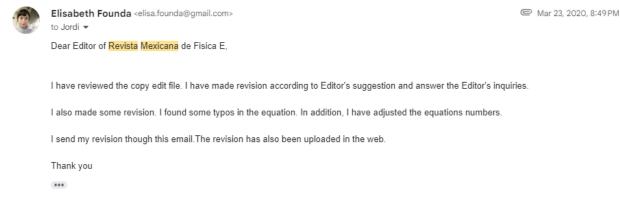
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Rev. Mex. Fis. ?? (\*?\*) (????) ???-???

# 5. Bukti submit final revision

23 Maret 2020



# **Authors' Comments:**

- 1. Equation numbers need to be adjusted. (This version has been adjusted)
- 2. Citation numbers need to be adjusted. (This version has been adjusted)
- 3. My own revisions or adjustments are indicated by yellowhighlight
- 4. PACS has been added
- 5. This version has been revised according to the editor's suggestions. The revision is indicated by orange-color font.

# Relating Simple Harmonic Motion and Uniform Circular Motion with *Tracker*

Elisabeth Pratidhina<sup>1, 2\*</sup>, Ferina Rizky Yuliani<sup>2</sup>, Wipsar Sunu Brams Dwandaru<sup>1</sup>

<sup>1</sup>Graduate School of Educational Science, Yogyakarta State University

<sup>2</sup>Department of Physics Education, Widya Mandala Catholic University Surabaya

\*Corresponding Email: elisa.founda@gmail.com

# ABSTRACT

In this study, we demonstrate an interesting relationship between the simple harmonic motion and uniform circular motion trough a simple experiment. The experiment requires a low cost, easily-found materials and free software Tracker. To represent uniform circular motion, we use a tape that is stuck on a fan moving with the constant angular speed. Meanwhile, spring and pendulum motion are used to represent simple harmonic motion. Through Video Tracker analysis, we have shown that the positions (x and y coordinates) of an object undergoes uniform circular motion fit to the sinusoidal function of time, similar to simple harmonic motion and uniform circular motion. This simple experiment can be used in high school physics courses to lead students in developing a conceptual understanding of uniform circular motion with a less mathematical approach.

# Keyword: simple harmonic motion, uniform circular motion, physics demonstration, Tracker

# PACS: 01.40.-d; 01.50.-i

### **INTRODUCTION**

Recently, there is a rapid development of imaging technology. Moreover, technology in mobile phones has been improved significantly. Nowadays, mobile phones provide various features such as cameras, internet, a variety of sensors, spreadsheets, games, and so on. The development of imaging technology and the inclusion of cameras on mobile phones can be used for physics teaching and learning activities. By using imaging technology, Tracker has become a useful tool in physics education. Tracker is free software developed on the Open Source Physics Java code library which can be used for video analysis and physics modeling. Tracker includes features such as object tracking with position, velocity, and acceleration overlays and graphs, calibration points and line profiles for analysis of spectra and interference patterns [1].

Some studies have explored the potential of Tracker for pedagogical learning tools [2-5]. Poonyawatpornkul and Wattanakasiwich [3] use Tracker to analyze damped harmonic oscillation; Rodrigues and Simeão Carvalho [6] use Tracker to teach optical phenomena; Wee, Tan, Leong, and Tan [5] integrate Tracker in physics lessons when discussing free-fall motion. Tracker allows students to identify relationships between physical quantities through observation, compare the real world with mathematical modeling, and be trained to construct a model for a particular physical phenomenon [7]

Circular motion and simple harmonic motion are discussed several times in high school physics and introductory college physics courses. Uniform circular motion and simple harmonic motion have close relationships. Simple harmonic motion is a term used when the periodic motion is a sinusoidal function of time. Most students are able to notice simple harmonic motion in the springmass system and the simple pendulum. In fact, the x- and y-coordinates of particles' positions in uniform circular motion are also a sinusoidal function of time, similar to simple harmonic motion. Sometimes students face difficulty to relate uniform circular motion and simple harmonic motion trough geometric and mathematical approaches.

In this work, we show a simple experiment using Tracker to compare uniform circular motion and simple harmonic motion. Experiments, representing uniform circular motion and simple harmonic motion, have been conducted with simple materials and tools which can be found easily in daily life. The phenomena were recorded with a mobile phone camera. After that, the videos were analyzed through Tracker.

# THEORY

Simple Harmonic Motion in Vertical Spring-Mass System

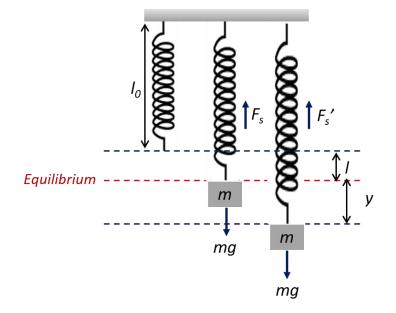


Figure 1. Vertical Spring-Mass System

One of the physical systems representing simple harmonic motion is a vertical spring-mass system. A particle with mass m is attached on vertical spring with an original length of  $l_o$  such as shown in Fig. 1. At equilibrium point, the spring force is balanced with the gravitational force, or according to Newton's first law,  $F_s + F_g = 0$ . Thus,

$$-kl + mg = 0$$

Or

$$kl = mg \tag{1}$$

When the spring is stretched down leaving its equilibrium position, there is acceleration, so that  $\Sigma F = ma$ ,

$$F'_{s} + F_{g} = ma,$$

$$-k(l+y) + mg = m\frac{d^{2}y}{dt^{2}}.$$
(2)

Since kl = mg,

$$-ky = m\frac{d^2y}{dt^2},\tag{3}$$

Eq. (3) can be written as:

$$\frac{d^2y}{dt^2} = -\omega^2 y \tag{4}$$

where  $\omega^2 = \frac{k}{m}$ .

By solving the second-order differential equation in Eq. (4), we yield a solution for the particle's position as follows:

$$y(t) = A\cos(\omega t + \phi), \tag{5}$$

where A is the amplitude and  $\phi$  is the initial phase. Both of A and  $\phi$  depend on the initial condition of the particle. As described in Eq. (5), the particle's periodic motion is a sinusoidal function of time, in which the sinusoidal function is a cosine function. It is the main characteristic of simple harmonic motion. Furthermore, we can determine the velocity and acceleration of the particle by taking the first and second derivative of Eq. (5), respectively. Thus,

$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi)$$
(6)

$$a(t) = \frac{d^2 y(t)}{dt^2} = \frac{dv(t)}{dt} = -A \,\omega^2 \cos(\omega t + \phi) \tag{7}$$

# **Uniform Circular Motion**

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t \tag{8}$$

where  $\omega$  is angular speed, whereas  $\phi$  is the initial angular position or initial phase.

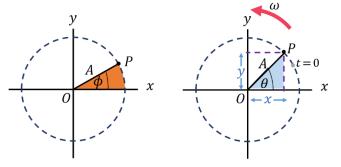


Figure 2. Particle *P* goes through the uniform circular motion with a constant angular speed of  $\omega$ . The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time *t*.

Particle *P* in Fig. 2 is moving in a uniform circular motion. It moves in a circle with a radius of *A* and constant angular speed  $\omega$ . The *x*-coordinate of particle P is given by

$$x(t) = A\cos\theta = A\cos(\omega t + \phi)$$
(9)

The x-coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The x-component of velocity and acceleration are given in Eqs. (10) and (11), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi)$$
(10)

$$a_x(t) = \frac{d}{dt} v_x(t) = -A\omega^2 \cos(\omega t + \phi)$$
(11)

The y-coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see Eq. (12)).

$$y(t) = \frac{A \sin \theta}{A \sin (\omega t + \phi)}$$
(12)

$$v_{y}(t) = \frac{d}{dt}y(t) = \frac{A\omega\cos(\omega t + \phi)}{(13)}$$

$$a_{y}(t) = \frac{d}{dt}v_{y}(t) = -A\omega^{2}\sin(\omega t + \phi)$$
(14)

# **EXPERIMENTAL SETUP**

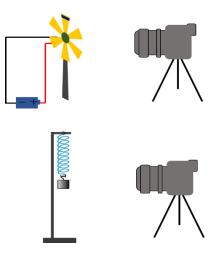


Figure 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.

The experimental setup is simple, as shown in Fig. 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Fig. 3a). The recorded video is then analyzed with Tracker. Through Tracker, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Fig. 3b).

## **RESULT AND DISCUSSION**

The position of an object attached to a fan which moves with constant angular speed has been analyzed through Tracker. As time goes on, the x-coordinate of the object goes down and up following cosine function (see Fig. 4) as predicted in theory. Although it is not perfectly smooth, the x-component of linear velocity tends to follow sine function with an initial phase ( $\phi$ ) of  $\pi$ radians. Meanwhile, the x-component of acceleration tends to follow negative cosine function, consistent with Eq. (11).

As shown in Fig. 5, the y-coordinate of the object on uniform circular motion smoothly fits with the sine function in Eq. (12). On the other hand, the y-component of velocity and acceleration graph are not as smooth as the position graph. However, the y-component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

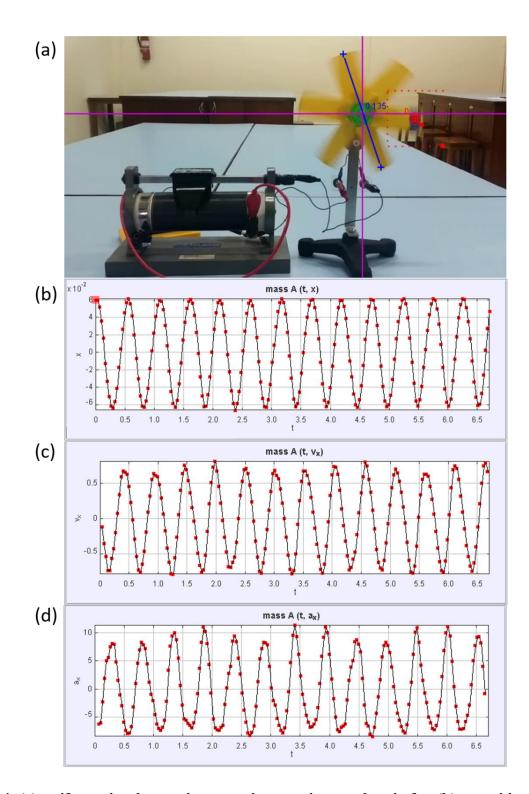


Figure 4. (a) uniform circular motion experiment using an electric fan (b) x-position versus time graph, (d), x-component linear velocity versus time graph, (e) x-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

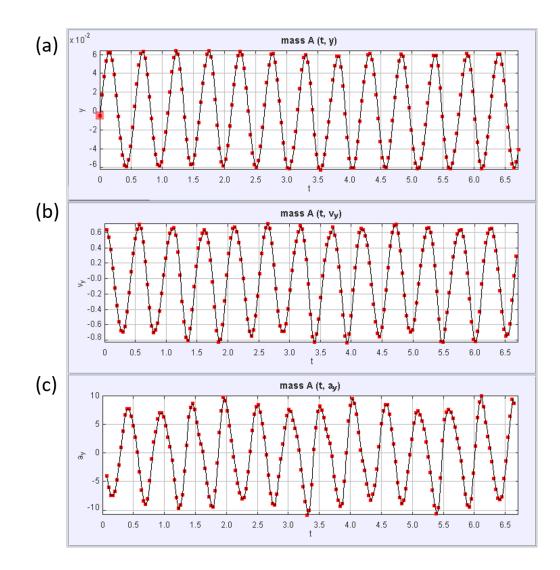


Figure 5. (a) *y*-position versus time graph, (c), *y*-component linear velocity versus time graph, (d) *y*-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

In addition, we performed an experiment with a well-known simple harmonic motion system, i.e., mass-spring system and simple pendulum. Fig. 6b-d shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ , whereas the acceleration fits cosine function or sine function with  $\phi = \frac{\pi}{2}$ .

The analysis of simple pendulum motion is presented in Fig. 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The ball was swung to the left initially. The position of the ball perfectly follows the sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\frac{\pi}{2}$ . Meanwhile, the acceleration graph fits the sine function with  $\phi = \frac{\pi}{2}$ .

We have shown through analysis with Tracker that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

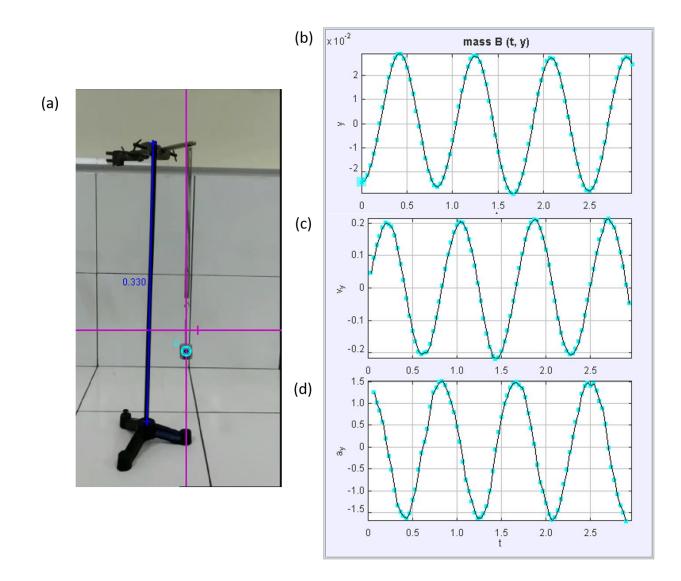


Figure 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) *y*-position, (c) *y*-component velocity, (d) *y*-component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI unit.

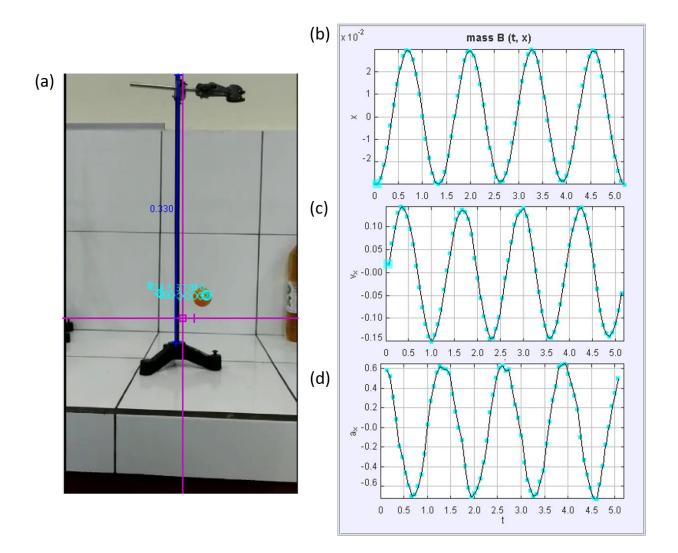


Figure 7. (a) simple pendulum experiment, (b) *x*-position, (c) *x*-component velocity, (d) *x*component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI unit.

# STRATEGIC IMPLEMENTATION IN CLASS

A simple experiment with Tracker could potentially be included in physics classroom to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with Tracker. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as project, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

# CONCLUSION

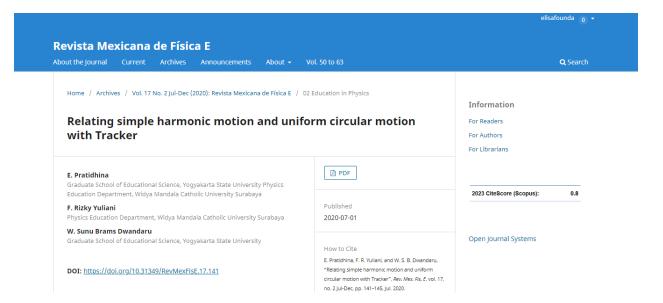
In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using Tracker with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The x- and y-coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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# 6. Bukti published online

# 1 Juli 2020



EDUCATION

### Relating simple harmonic motion and uniform circular motion with Tracker

E. Pratidhina<sup>a,b,\*</sup>, F. Rizky Yuliani<sup>b</sup>, and W. Sunu Brams Dwandaru<sup>a</sup> <sup>a</sup>Graduate School of Educational Science, Yogyakarta State University. <sup>b</sup>Department of Physics Education, Widya Mandala Catholic University Surabaya. \*e-mail: elisa.founda@gmail.com

Received 31 January 2020; accepted 18 February 2020

In this study, we demonstrate an interesting relationship between the simple harmonic motion and uniform circular motion trough a simple experiment. The experiment requires a low cost, easily-found materials, and free software Tracker. To represent uniform circular motion, we use a tape that is stuck on a fan moving with the constant angular speed. Meanwhile, spring and pendulum motion are used to represent simple harmonic motion. Through video analysis with Tracker, we have shown that the positions (*x* and *y* coordinates) of an object undergoes uniform circular motion fit to the sinusoidal function of time, similar to simple harmonic motion. We also analyze the behavior of velocity and acceleration in simple harmonic motion and uniform circular motion. This simple experiment can be used in high school physics courses to lead students in developing a conceptual understanding of uniform circular motion with a less mathematical approach.

Keywords: Simple harmonic motion; uniform circular motion; physics demonstration; Tracker

PACS: 01.40.-D; 01.50.-I

#### 1. Introduction

Recently, there is a rapid development of imaging technology. Moreover, technology in mobile phones has been improved significantly. Nowadays, mobile phones provide various features such as cameras, internet, a variety of sensors, spreadsheets, games, and so on. The development of imaging technology and the inclusion of cameras on mobile phones can be used for physics teaching and learning activities. By using imaging technology, Tracker has become a useful tool in physics education. Tracker is a free software developed on the Open Source Physics Java code library which can be used for video analysis and physics modeling. Tracker includes features such as object tracking with position, velocity, and acceleration overlays and graphs, calibration points and line profiles for analysis of spectra and interference patterns [1].

Some studies have explored the potential of Tracker for pedagogical learning tools [2-5]. Poonyawatpornkul and Wattanakasiwich [3] use Tracker to analyze damped harmonic oscillation; Rodrigues and Simeão Carvalho [6] use Tracker to teach optical phenomena; Wee, Tan, Leong, and Tan [5] integrate Tracker in physics class when discussing free-fall motion. Tracker allows students to identify relationships between physical quantities through observation, compare the real world with mathematical modeling, and be trained to construct a model for a particular physical phenomenon [7].

Circular motion and simple harmonic motion are discussed several times in high school physics and introductory college physics courses. Uniform circular motion and simple harmonic motion have close relationships. Simple harmonic motion is a term used when the periodic motion is a sinusoidal function of time. Most students are able to notice simple harmonic motion in the spring-mass system and the simple pendulum. In fact, the x- and y-coordinates of particles' positions in uniform circular motion are also a siDOI: https://doi.org/10.31349/RevMexFisE.17.141

nusoidal function of time, similar to simple harmonic motion. Sometimes students face difficulty to relate uniform circular motion and simple harmonic motion trough geometric and mathematical approaches.

In this work, we show a simple experiment using Tracker to compare uniform circular motion and simple harmonic motion. Experiments, representing uniform circular motion and simple harmonic motion, have been conducted with simple materials and tools which can be found easily in daily life. The phenomena were recorded with a mobile phone camera. After that, the videos were analyzed through Tracker.

#### 2. Theory

# 2.1. Simple harmonic motion in vertical spring-mass system

One of the physical systems representing simple harmonic motion is a vertical spring-mass system. A particle with mass m is attached on vertical spring with an original length of  $l_o$  such as shown in Fig. 1. At equilibrium point, the spring force is balanced with the gravitational force, or according to Newton's first law,  $F_s + F_q = 0$ . Thus,

$$-kl + mg = 0,$$

or

$$= mg.$$
 (1)

When the spring is stretched down leaving its equilibrium position, there is acceleration, so that  $\sum F = ma$ 

kl

$$F_s + F_g = ma,$$
  
$$-k(l+y) + mg = m\frac{d^2y}{dt^2}.$$
 (2)

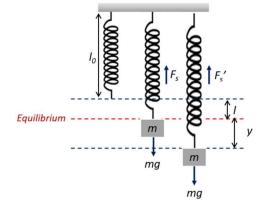


FIGURE 1. Vertical Spring-Mass System.

Since kl = mg,

$$-ky = m\frac{d^2y}{dt^2}.$$
(3)

Eq. (3) can be written as:

$$\frac{d^2y}{dt^2} = \omega^2 y, \tag{4}$$

where  $\omega^2 = k/m$ .

By solving the second-order differential equation in Eq. (4); we yield a solution for the particle's position as follows:

$$y(t) = A\cos(\omega t + \phi) \tag{5}$$

where A is the amplitude and  $\phi$  is the initial phase. Both of A and  $\phi$  depend on the initial condition of the particle. As described in Eq. (5), the particle's periodic motion is a sinusoidal function of time, in which the sinusoidal function is a cosine function. It is the main characteristic of simple harmonic motion. Furthermore, we can determine the velocity and acceleration of the particle by taking the first and second derivative of Eq. (5), respectively. Thus,

$$v(t) = \frac{dy(t)}{dt} = -A\omega\sin(\omega t + \phi),$$
(6)

$$a(t) = \frac{dv(t)}{dt} = -A\omega^2 \cos(\omega t + \phi).$$
(7)

### 2.2. Uniform circular motion

A particle moves within uniform circular motion when it moves in a circular path with a constant angular speed. The angular position of a particle is given as:

$$\theta(t) = \phi + \omega t,\tag{8}$$

where  $\omega$  is angular speed, whereas  $\phi$  is the initial angular position or initial phase.

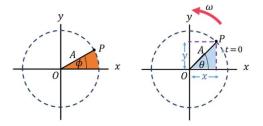


FIGURE 2. Particle P goes through the uniform circular motion with a constant angular speed of  $\omega$ . The figure on the left is the initial particle's position. The figure on the right side is the particle's position at time t.

Particle P in Fig. 2 is moving in a uniform circular motion. It moves in a circle with a radius of A and constant angular speed  $\omega$ . The x-coordinate of particle P is given by

$$x(t) = A\cos\theta = A\cos(\omega t + \phi).$$
(9)

The x-coordinate of particle P is a sinusoidal function of time, analogous to the simple harmonic motion. The x-component of velocity and acceleration are given in Eqs. (10) and (11), respectively.

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega t\sin(\omega t + \phi)$$
(10)

$$a_x(t) = \frac{d}{dt}v_x(t) = -A\omega^2 t\cos(\omega t + \phi)$$
(11)

The y-coordinates of particle P is also analogous to SHM motion, following the sinusoidal function of time (see Eq. (12)).

$$y(t) = A\sin\theta = A\sin(\omega t + \phi), \tag{12}$$

$$v_x(t) = \frac{d}{dt}y(t) = A\omega\cos(\omega t + \phi), \qquad (13)$$

$$a_y(t) = \frac{d}{dt}v_y(t) = -A\omega^2 t\sin(\omega t + \phi).$$
(14)

### 3. Experimental setup

The experimental setup is simple, as shown in Fig. 3. The behavior of uniform circular motion is investigated by recording the motion of tape, which is stuck on an electric fan (see Fig. 3a). The recorded video is then analyzed with Tracker. Through Tracker, we get data about position, velocity, and acceleration as a function of time. Meanwhile, the behavior of simple harmonic motion is represented by the motion of a slotted weight which is attached to a vertical spring (see Fig. 3b).

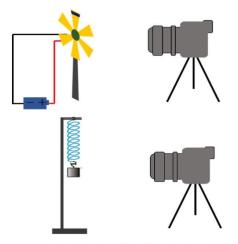


FIGURE 3. Experimental set-up of (a) uniform circular motion using an electrical fan, (b) simple harmonic motion using spring and slotted weight.

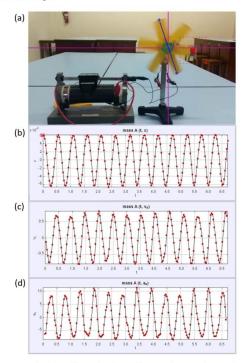


FIGURE 4. (a) Uniform circular motion experiment using an electric fan, (b) x-position versus time graph, (c) x-component linear velocity versus time graph, (d) x-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

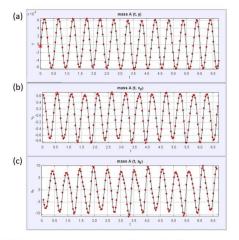


FIGURE 5. (a) y-position versus time graph, (b) y-component linear velocity versus time graph, (c) y-component acceleration versus time graph in a uniform circular motion. All quantities are presented in SI units.

### 4. Result and discussion

The position of an object attached to a fan which moves with constant angular speed has been analyzed through Tracker. As time goes on, the *x*-coordinate of the object goes down and up following cosine function (see Fig. 4b) as predicted in theory. Although it is not perfectly smooth, the *x*-component of linear velocity tends to follow sine function with an initial phase ( $\phi$ ) of  $\pi$  radians. Meanwhile, the *x*-component of acceleration tends to follow negative cosine function, consistent with Eq. (11).

As shown in Fig. 5, the y-coordinate of the object on uniform circular motion smoothly fits with the sine function in Eq. (12). On the other hand, the y-component of velocity and acceleration graph are not as smooth as the position graph. However, the y-component of velocity and acceleration still tend to follow cosine and negative sine function, respectively. These results demonstrate that uniform circular motion has symmetry with simple harmonic motion.

In addition, we performed an experiment with a wellknown simple harmonic motion system, *i.e.* mass-spring system and simple pendulum. Figure 6b-d) shows the position, velocity, and acceleration of a slotted weight attached on a vertical spring as functions of time. The spring was initially stretched down. As predicted, the position of slotted weight fits sine function with  $\phi = -\pi$  radians. The velocity fits cosine function with  $\phi = -\pi/2$ , whereas the acceleration fits cosine function or sine function with  $\phi = \pi/2$ .

The analysis of simple pendulum motion is presented in Fig. 7. Consistent with the mass-spring system, the periodic motion of the pendulum is a sinusoidal function of time. The

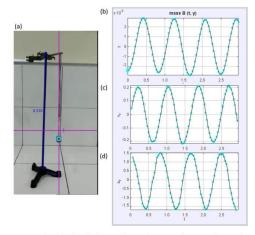


FIGURE 6. (a) simple harmonic motion experiment using spring and slotted weight, (b) y-position, (c) y-component velocity, (d) y-component acceleration of mass as functions of time in the simple harmonic motion of a spring-mass system. All quantities are presented in SI units.

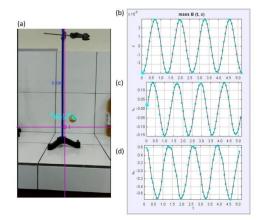


FIGURE 7. (a) simple pendulum experiment, (b) x-position, (c) x-component velocity, (d) x-component acceleration of mass as functions of time in a simple harmonic motion of simple pendulum system. All quantities are presented in SI units.

ball was swung to the left initially. The position of the ball perfectly follows the sine function with  $\phi = \pi$  radians. The velocity fits cosine function with  $\phi = -\pi/2$ . Meanwhile, the acceleration graph fits the sine function with  $\phi = \pi/2$ .

We have shown through analysis with Tracker that uniform circular motion has symmetry with simple harmonic motion. The linear position, velocity, and acceleration in uniform circular motion fit the sinusoidal function of time, which is the same in simple harmonic motion.

#### 5. Strategic implementation in class

A simple experiment with Tracker could potentially be included in physics classrooms to enhance students' conceptual understanding in a uniform circular motion and simple harmonic motion. There are some possible ways to implement this simple experiment in the classroom. For example, the teacher can bring a recorded video about uniform circular motion and simple harmonic motion in class. At the beginning of the class, the videos are distributed to students. With teacher guidance, students are asked to analyze the videos with Tracker. In the scenario, students, individually or in a group, can record the uniform circular motion and simple harmonic motion by themselves with the teacher's help. This scenario will engage students more in the experiments and hopefully, students become more motivated in learning physics.

The teacher can make this activity as project, laboratory activity, homework or class demonstration. After the activity, the teacher can discuss the relation between simple harmonic motion and uniform circular motion in class through a mathematical approach. The discussion will enrich the students' conceptual knowledge about simple harmonic motion and circular motion.

### 6. Conclusion

In summary, we have demonstrated the relationship between uniform circular motion and simple harmonic motion by a simple experiment using Tracker with non-mathematical approach. The equation of motion of an object which moves in a uniform circular motion is analog to an object which moves in simple harmonic motion such as in spring-mass or pendulum system. The x- and y-coordinates of an object undergoes uniform circular motion, fit to the sinusoidal function of time. The velocity and acceleration also fit the sinusoidal function of time. In other words, we can say that simple harmonic motion can be considered as a projection of uniform circular motion. This demonstration can be embedded in high school physics classes to enhance students' conceptual understanding of simple harmonic motion and uniform circular motion.

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