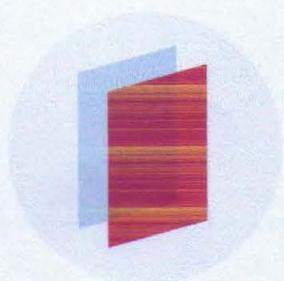


PAPER • OPEN ACCESS

## Preface/Introduction

To cite this article: 2019 *J. Phys.: Conf. Ser.* **1366** 011001

View the [article online](#) for updates and enhancements.



**IOP | ebooks™**

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

## Preface/Introduction



2nd International Conference on Applied & Industrial Mathematics and Statistics 2019  
(ICoAIMS 2019)  
*23-25th July 2019, The Zenith Hotel, Kuantan, Pahang, Malaysia.*

2nd International Conference on Applied & Industrial Mathematics and Statistics 2019 (ICoAIMS 2019) is organised by Faculty of Industrial Sciences & Technology, Universiti Malaysia Pahang, Malaysia. Our co-organisers are Institut Teknologi Sepuluh (ITS) Nopember, Surabaya, Indonesia, Malaysian Mathematical Sciences Society (PERSAMA) and Kazakh National Agrarian University, Kazakhstan. The main topics of the conference is divided into six categories; Pure Mathematics, Applied Mathematics, Computational Mathematics, Statistics & Applied Statistics, Operational Research and Mathematics Education including Engineering & Industrial Applications.

The ICoAIMS 2019 with the theme *IR 4.0 Through the Eyes of Mathematics* aims to bring together leading academics, scientists, researchers and research scholars to exchange and share their experiences and research results on all aspects related to Mathematics and Statistics. It also provides a premier interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, trends, and concerns as well as practical challenges encountered and solutions adopted in the fields of Mathematics.

ICoAIMS 2019 was an overwhelming success, attracting the delegates, speakers and sponsors from many countries and provided great intellectual and social interaction for the



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

participants. Without their support, the conference would not have been the success that it was. We trust that all the participants found their involvement in the Conference both valuable and rewarding. Once again, we would like to convey our deepest appreciation for all contributions and wish you success in the years ahead.

#### Editors

Dr. Nor Izzati Jaini

Dr. Norazaliza Mohd Jamil

Dr. Anvarjon Ahmedov Ahat Jonovich

Dr. Abdul Rahman Mohd Kasim

Mrs Siti Fatimah Ahmad Zabidi

Mrs Rahimah Jusoh @ Awang

#### Conference/Sponsor Logo



### Organising Committee

**Chairman:**

Associate Professor Dr. Mohd Zuki Salleh

**Co-Chairman:**

Dr. Zulkhibri Ismail

**Secretary**

Dr Norhayati Rosli

**Treasurer:**

Dr. Muhammad Azrin Ahmad

Dr. Nor Aida Zuraimi Md Noar

**Sponsorship:**

Miss Rozieana Khairuddin

Dr. Zulkhibri Ismail

Miss Nur Zahirah Md Noor

**Logistic:**

Mrs Ezrinda Mohd Zaihidee

Dr. Yuhani Yusof

Mr Muhammad Halim Paboh

**Keynote/Invited Speaker:**

Dr. Noryanti Muhammad

**Program Book:**

Dr. Norhafizah Md Sarif

Mrs Najihah Mohamed

**Proceedings & Journals Publication:**

Dr. Norazaliza Mohd Jamil

Dr. Anvarjon Ahmedov Ahat Jonovich

Dr. Nor Izzati Jaini

Dr. Abdul Rahman Mohd Kasim

Mrs Siti Fatimah Ahmad Zabidi

Mrs Rahimah Jusoh @ Awang

**Registration & Souvenirs:**

Miss Nurfatihah Mohamad Hanafi

Mrs Intan Sabariah Sabri

**Protocol:**

Ts. Dr. Azlyna Senawi

Dr Siti Roslindar Binti Yaziz

**Publicity & Website:**

Dr. Nor Alisa Mohd Damanhuri

Miss Laila Amera Aziz

### Scientific Committee

- Assoc. Prof. Dr. Dumitru Vieru, *Gheorghe Asachi Technical University of Iasi, Romania*  
Emeritus Prof. Constantin Fetecau, *Gheorghe Asachi Technical University of Iasi, Romania*  
Prof. Frank Coolen, *University of Durham, UK*  
Prof. Qi Wang, *University of South Carolina, US*  
Prof. Xuerong Mao, *University of Strathclyde, United Kingdom*  
Prof. Dr. Ioan Pop, *Babeş-Bolyai University*  
Dr. John Boland, *University of South Australia, Australia*  
Emeritus Prof. Phil Howlett, *University of South Australia, Australia*  
Prof. Serikbayev Abdulkarim, *National Algharian University, Kazakhstan*  
Dr Sergey Utyuzhnikov, *University of Manchester, United Kingdom*  
Prof. Dr. Basuki Widodo, *Sepuluh Nopember Institute of Technology, Indonesia*  
Assoc. Prof. Dr. Heri Kuswanto, *Sepuluh Nopember Institute of Technology, Indonesia*  
Dr. Imam Mukhlash, *Sepuluh Nopember Institute of Technology, Indonesia*  
Dr. Suhartono, *Sepuluh Nopember Institute of Technology, Indonesia*  
Prof. Dr. Shaharuddin Salleh, *Universiti Teknologi Malaysia*  
Prof. Dr. Muhammad Hisyam Lee, *Universiti Teknologi Malaysia*  
Dr. Yeak Su Hoe, *Universiti Teknologi Malaysia*  
Prof. Maslina Darus, *Universiti Kebangsaan Malaysia*  
Prof Dr Kamarulzaman Ibrahim, *Universiti Kebangsaan Malaysia*  
Assoc. Prof. Dr. Maznah Mat Kasim, *Universiti Utara Malaysia*  
Assoc. Prof. Dr. Mohd Kamal Mohamad Nawawi, *Universiti Utara Malaysia*  
Assoc. Prof. Dr. Mohd Rashid Ab Hamid, *Universiti Malaysia Pahang*  
Assoc. Prof. Dr. Anvarjon Ahmedov Ahatjonovich , *Universiti Malaysia Pahang*  
Dr. Abdul Rahman Mohd Kasim, *Universiti Malaysia Pahang*  
Dr. Roslinazairimah Zakaria, *Universiti Malaysia Pahang*  
Dr. Nor Aida Zuraimi Md Noar, *Universiti Malaysia Pahang*  
Dr. Mohd Sham Mohamad, *Universiti Malaysia Pahang*

### Keynote Speakers

- Dato' Sri. Dr. Mohd Uzir Mahidin  
*Chief Statistician, Department of Statistics Malaysia*
- Emeritus Prof. Dr. Ioan Pop  
*Faculty of Mathematics and Computer Science, University of Babeş-Bolyai, Romania*
- Prof Kalmenov Tynysbek Sharipovich  
*National Academy of Sciences, Republic of Kazakhstan*
- Prof Dr. Mohd Isa Irawan  
*Faculty of Mathematics Computing and Data Science (FMKSD), Institut Teknologi Sepuluh Nopember Surabaya, Indonesia*
- Dr Dzaharudin Mansor  
*National Technology Officer, Microsoft Malaysia*
- Dr Suhartono  
*Department of Statistics, Institut Teknologi Sepuluh Nopember (ITS) Surabaya, Indonesia*

This site uses cookies. By continuing to use this site you agree to our use of cookies. To find out more, see our [Privacy and Cookies policy](#).



## Table of contents

Volume 1366

2019

[◀ Previous issue](#)   [Next issue ▶](#)

**2nd International Conference on Applied & Industrial Mathematics and Statistics  
23–25 July 2019, Kuantan, Pahang, Malaysia**

[View all abstracts](#)

**Accepted papers received: 8 October 2019**

**Published online: 7 November 2019**

### Preface

OPEN ACCESS 011001

Preface/Introduction

[+ View abstract](#)   [View article](#)   [PDF](#)

OPEN ACCESS 011002

Peer review statement

[+ View abstract](#)   [View article](#)   [PDF](#)

### Papers

#### Applied Mathematics

OPEN ACCESS 012001

**Substitution Box Design Based from Symmetric Group Composition**

Muhammad Fahim Bin Roslan, Kamaruzzaman Seman, Azni Haslizan Ab Halim and M Nor Azizi Syam  
Mohd Sayuti

[+ View abstract](#)   [View article](#)   [PDF](#)

OPEN ACCESS 012002

**OPEN ACCESS**

012053

**Comparison of surgically induced astigmatism (SIA) values using three Holladay incorporated method SIA calculators**

M M Md Muziman Syah, M Nurul Adabiah, A H Noorhazayti, M Nazaryna, M Azuwan, M Noryanti, C A Mohd Zulfaezal and B Noor Ezailina

[+ View abstract](#) [View article](#) [PDF](#)**OPEN ACCESS**

012054

**Kinematics study on PLF technique by comparing professional and amateur Malaysian army parachutists based on event during landing**

S Aziz, A S Rambely and U F A Rauf

[+ View abstract](#) [View article](#) [PDF](#)**Pure Mathematics****OPEN ACCESS**

012055

**Maclaurin Heat Coefficients and Associated Zeta Functions on Quaternionic Projective Spaces  $P^n(\mathbb{H})$  ( $n \geq 1$ )**

Richard Olu Awonusika

[+ View abstract](#) [View article](#) [PDF](#)**OPEN ACCESS**

012056

**Some algebraic Rhotrices using a method of spanning**

Muhammad Hassan Muhammad

[+ View abstract](#) [View article](#) [PDF](#)**OPEN ACCESS**

012057

**Finitely Memory Strategies in Special Büchi Games with Inductive Measurable Payoffs**

Ahmad Termimi Ab Ghani

[+ View abstract](#) [View article](#) [PDF](#)**OPEN ACCESS**

012058

**The implementation of z-numbers in fuzzy clustering algorithm for wellness of chronic kidney disease patients**

N J Mohd Jamal, K M N Ku Khalif and M S Mohamad

[+ View abstract](#) [View article](#) [PDF](#)**OPEN ACCESS**

012059

**Second Hankel determinant for bounded turning functions of order beta of certain subclasses of analytic functions**

A Yahya and M N Tokachil

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012060

Existence of immovability lines of a partial mapping of Euclidean space  $E_5$ 

Gulbadan Matieva, Cholpon Abdullayeva and Anvarjon Ahmedov

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012061

About existence of quasi-double lines of the partial mapping of space  $E_n$ 

Gulbadan Matieva, Cholpon Abdullayeva and Anvarjon Ahmedov

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012062

## Algebraic operations on new interval neutrosophic vague sets

Hazwani Hashim, Lazim Abdullah and Ashraf Al-Quran

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012063

A Note of The Linear Equation  $AX = B$  with Multiplicatively-Reguler Matrix  $A$  in Semiring

Gregoria Ariyanti

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012064

Subgraph of Compatible Action Graph for Finite Cyclic Groups of  $p$ -Power Order

Mohammed Khalid Shahoodh, Mohd Sham Mohamad, Yuhani Yusof and Sahimel Azwal Sulaiman

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012065

## On solvability of some boundary value problems for the non-local polyharmonic equation with boundary operators of the Hadamard Type

Batirkhan Turmetov, Moldir Muratbekova and Anvarjon Ahmedov

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012066

## Splicing System in Automata Theory : A Review

S H Khairuddin, M A Ahmad and N Adzhar

[+ View abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012067

## On the generalized Radimacher-Menchoff Theorem for general spectral decomposition of the elliptic differential operators

Anvarjon Ahmedov, Ehab Matarneh and Mohammad Hasan bin Abd Sathar

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012068

## Transformation of the Mean Value of Integral On Fourier Series Expansion

Gani Gunawan, Erwin Harahap and Suwanda

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012069

On the relation between *CT*-Groups and *NSP*-Groups on finite Groups

Khaled mustafa Al-Jamal and Ahmad Termimi Ab Ghani

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012070

## On Generalized Derivations of some classes of finite dimensional algebras

Sh. K. Said Husain, W. Basri and A. Abdulkadir

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012071

## Semi Bornological Groups

Anwar N. Imran and Sh. K. Said Husain

[+ View abstract](#) [View article](#) [PDF](#)**Operational Research**

## OPEN ACCESS

012072

## Optimal Design of a Rain Gauge Network Models: Review Paper

Nor Sofiza Abu Salleh, Mohd Khairul Bazli Mohd Aziz and Noraziah Adzhar

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012073

## Solving the Goods Transportation Problem Using Genetic Algorithm with Nearest-Node Pairing Crossover Operator

Amirah Rahman, Nazmi Syazwan Shahrudin and Ismail Ishak

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012074

## Estimation of Maximum Sustainable Yield (MSY) for Sustainable Fish Catch

N A Jamaluddin, S A Sheikh Hussin, Z Zahid and S S Mohd Khairi

[+ View abstract](#) [View article](#) [PDF](#)

## OPEN ACCESS

012075

## A Bootstrap Simulation for comparison of Group Risk Plan and Multi-Peril Crop Insurance Policy

Valantino Agus Sutomo, Dian Kusumaningrum, Rahma Anisa and Aryana Paramita

PAPER • OPEN ACCESS

## A Note of The Linear Equation $AX = B$ with Multiplicatively-Regular Matrix $A$ in Semiring

To cite this article: Gregoria Ariyanti 2019 *J. Phys.: Conf. Ser.* **1366** 012063

View the [article online](#) for updates and enhancements.

You may also like

- [On Soft Ternary -Semirings-I](#)  
B. Ravi Kumar, B. Sankara Rao, D. Madhusudhana Rao et al.
- [Sensing analysis based on fano resonance in arch bridge structure](#)  
Lirong Dong, Xuemei Xu, Kehui Sun et al.
- [Commutativity of Prime -Semirings with Derivations and Generalized Derivations](#)  
Shahed A. Hamil



The Electrochemical Society  
Advancing solid state & electrochemical science & technology

243rd Meeting with SOFC-XVIII

Boston, MA • May 28 – June 2, 2023

Accelerate scientific discovery!

Learn More & Register



# A Note of The Linear Equation $AX = B$ with Multiplicatively-Regular Matrix $A$ in Semiring

**Gregoria Ariyanti**

Department of Mathematics Education, Faculty of Teacher Training and Education  
Catholic, University of Widya Mandala, Madiun 63131 Madiun, Indonesia

E-mail: [ariyantigregoria@gmail.com](mailto:ariyantigregoria@gmail.com)

**Abstract.** Semiring is a form of generalization of the ring, where one or more conditions in the ring are removed. An element  $a$  is called multiplicatively-regular if there is  $x$  so  $axa = a$ . In real number algebra, a system of linear equations  $AX = B$  has a singular solution if a matrix  $A$  has an inverse. Elements of semiring which does not a zero element have no inverse of addition. By reviewing matrix  $A$  as a multiplicatively-regular, it is develop of necessary or sufficient condition of semiring. Given a matrix  $A$  with the right complement matrix  $A^r$  satisfies  $AA^r = 0$ . The sufficient condition of the linear equations system  $AX = B$  has a solution is there exist a matrix  $B$  satisfies  $AA^\circ B = B$  and a matrix  $A$  has a right complement matrix  $A^r$ .

## 1. Introduction

A non-empty set  $G$  with a binary operation  $*$  is called Group if it has the following properties: associative, has an identity element of binary operations  $*$ , and every element that is not an identity element has an inverse. Meanwhile, a non-empty set  $R$  with two binary operations namely  $*$  and  $\circ$  is called Ring if it has the following properties:  $(R, *)$  is a commutative group, closed to binary operation  $\circ$ , associative to binary operation  $\circ$ , and to both binary operations  $*$  and  $\circ$  is distributive. If the ring has the following properties: commutative to binary operation  $\circ$ , it has a unit element of binary operation  $\circ$ , and every element that is not a zero element has an inverse to binary operation  $\circ$ , it is called Field. If the conditions of a Group and Ring are weakened, other algebraic structures will appear, namely Semigrup and Semiring. That is, if some Group or Ring conditions are removed, the algebraic structure formed is Semigrup and then Semiring [2,6]. One of the problems and applications that are often encountered in mathematics is to complete the Linear Equations System [2,5].

The linear equations system that has been developed by researchers is a system of linear equations over Field which include real numbers  $\mathbb{R}$  or complex numbers  $\mathbb{C}$  [5]. In other studies, the object of research is extended not to Field anymore, but to commutative Ring and linear equations system Commutative over ring have been discussed by Brewer, et al. [3]. Likewise, assuming an extension from the Ring to Ring commutative does not change the definition in general [4].

In this paper, we show about matrix in linear equations system over semiring. In Section 2, we will review some basic facts for semiring, matrix over semiring, and the linear equations system over semiring. In Section 3, we show the results that is the necessary or sufficient condition of



solution of linear equations system over semiring by reviewing  $A$  as a multiplicatively-regular and a right complement matrix.

## 2. Some Preliminaries on Semiring

In this section, we review some basic facts for the semiring and a matrix over the semiring.

### 2.1. Semiring

**Definition 2.1.** ([6]) *Semigroup  $S$  is an empty set that is equipped with an associative binary  $*$  operation, that is for every  $x, y, z \in S, x * (y * z) = (x * y) * z$ .*

Furthermore, Poplin provides the following definition.

**Definition 2.2.** ([6]) *Semiring is a non-empty set  $S$  has two binary operations, addition  $(+)$  and multiplication  $(\times)$ , which has the following properties.*

- (i) *Operation  $+$  is commutative and associative, that is*
  - (a)  $a + b = b + a$  for every  $a, b \in S$ ,
  - (b)  $(a + b) + c = a + (b + c)$  for each  $a, b, c \in S$ .
- (ii) *Operation  $\times$  is associative and distributive to  $+$ , i.e.*
  - (a)  $(ab)c = a(bc)$  for each  $a, b, c \in S$
  - (b)  $a(b + c) = (ab) + (ac)$  for each  $a, b, c \in S$ ,
  - (c)  $(b + c)a = (ba) + (ca)$  for each  $a, b, c \in S$ .
- (iii) *The set  $S$  has a zero element  $0 \in S$  so that*
  - (a)  $0 + a = a + 0 = a$  for every  $a \in S$
  - (b)  $0 \times a = a \times 0 = 0$  for each  $a \in S$  and here in after called the absorbent element (absorption).
- (iv) *The set  $S$  has a unit element  $e, e \times a = a \times e = a$  for every  $a \in S$ .*

As in the Group and Ring structure, the commutative and idempotent nature also applies to certain Semiring. This is as stated by Poplin ([6]) below.

Poplin [6] stated if  $\times$  operation is commutative then  $S$  is called a commutative semiring and if  $+$  operation is idempoten then  $S$  is called a idempotent semiring.

### 2.2. Matrices over Semiring

We let  $M_{n \times 1}(S)$  is the set of all  $n \times 1$  vectors with elements from semiring  $S$ . We also let  $M_{n \times n}(S)$  is the set of all  $n \times n$  matrices with elements from semiring  $S$ . The  $+$  and  $\times$  operation for a matrices over semiring is defined as :

**Definition 2.3.** *Let  $S$  semiring, a positive integer  $n$  and  $M_n(S)$  is the set of all  $n \times n$  matrices over  $S$ . For every  $A, B \in M_n(S)$ ,  $+$  and  $\times$  operations over semiring  $S$  are defined :*

$$C = A + B \Rightarrow c_{ij} = a_{ij} + b_{ij}$$

$$C = A \times B \Rightarrow c_{ij} = \sum_l a_{il} \times b_{lj}$$

Therefore semiring has 0 as a zero element and 1 as an identity element, as in the matrix of conventional algebra, then a zero matrix and a identity matrix can be formed. The zero matrix  $n \times n$  over semiring  $S$  is  $0_n$  is defined as matrix with all elements equal to the 0–element, that is  $(0_n)_{ij} = 0$ . The identity matrix  $n \times n$  over  $S$  is defined as the matrix with all elements equal to the  $e$ –element, that is  $I_n$  with  $[I_n]_{ij} = \begin{cases} e, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$ .

### 2.3. The Linear Equations System over Semiring

We consider linear equations system over semirings  $S$  whose equations and variables are indexed by arbitrary sets, not necessarily ordered. In the following, if  $I, J$  and  $S$  are finite and non-empty sets then an  $I \times J$  matrix over  $S$  is a function  $A : I \times J \rightarrow S$ . An  $I$ -vector over  $S$  is defined similarly as a function  $b : I \rightarrow S$ . We write  $(A, b)$  as a matrix equation  $A \cdot x = b$ , where  $x$  is a  $J$ -vector of variables in  $S$ . The system  $(A, b)$  is said to be solvable if there exists a solution vector  $c : J \rightarrow S$  such that  $A \cdot c = b$ , where we define multiplication of unordered matrices and vectors in the usual way by  $(A \cdot c)(i) = \sum_{j \in J} A(i, j) \cdot c(j)$  for all  $i \in I$ . Similarly, a system of linear equations over a commutative semiring  $S$  is a pair  $(A, b)$  where  $A$  is an  $I \times J$  matrix with entries in  $S$  and  $b$  is an  $I$ -vector over  $S$ . The linear equations system can be expressed in other forms, as in the following description. We consider linear equations system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned} \quad (1)$$

The linear equations system (1) represented  $m$  linear equations with  $n$  arbitrary variable.

The matrix for the system of equations above are

$$Ax = B \quad (2)$$

with matrix  $A = (a_{ij}) \in M_{m \times n}(S)$ ,  $B = (b_1 \ b_2 \ \dots \ b_m)^T \in S^m$ , and  $x = (x_1 \ x_2 \ \dots \ x_m)^T \in S^m$  ([1]).

The equation in (1) or (2) stated had a solution in  $S^m$ , if there a vector  $\zeta \in S^m$  such that  $A\zeta = B$ . If  $B = 0$  then the linear equations system  $Ax = 0$  is called homogeneous system. The homogeneous system has at least one solution, that is  $\zeta = 0 = (0 \ 0 \ \dots \ 0)^T \in S^m$ . The solution  $\zeta = 0$  is called a trivial solution in the linear equations system  $Ax = 0$ . Furthermore, a vector  $\zeta \in S^m$  is called a non trivial solution in  $Ax = 0$  if  $\zeta \neq 0$  and  $A\zeta = 0$  ([5]).

## 3. The Results

### 3.1. Properties of Elements of Matrices Over Semiring

**Definition 3.1.** Given a semiring  $S$ . An  $a$  in  $S$  is called multiplicatively-regular element if and only if there exist  $a^*$  in  $S$  such that  $aa^*a = a$ .

Let  $a^\circ = a^*aa^*$ , we have

$$aa^\circ a = aa^*aa^*a = aa^*a = a$$

and

$$a^\circ aa^\circ = a^*aa^*aa^*aa^* = a^*aa^*aa^* = a^*aa^*a = a^\circ \quad (3)$$

From above definition, we can describe the following properties.

**Lemma 3.2.** Let  $S$  semiring and  $a \in S$ . If  $a$  is multiplicatively-regular element then  $a^\circ, a^\circ a$ , and  $aa^\circ$  are multiplicatively-regular element.

*Proof.* Let  $a$  is a multiplicatively-regular element and  $a^\circ = a^*aa^*$ . We have the following results:

- (i)  $a^\circ a^* a^\circ = a^*aa^*a^*a^*aa^* = a^*aa^*aa^* = a^*aa^* = a^\circ$ .
- (ii)  $(a^\circ a)a^*(a^\circ a) = (a^*aa^*a)a^*(a^*aa^*a) = a^*aa^*a^*a = a^*aa^*a = a^\circ a$ .
- (iii)  $(aa^\circ)a^*(aa^\circ) = aa^*aa^*a^*aa^*aa^* = aa^*aa^*aa^*aa^* = aa^*aa^\circ = aa^\circ$ .

Therefore,  $a^\circ, a^\circ a$ , and  $aa^\circ$  are multiplicatively-regular elements. □

From the set of all multiplicatively-regular elements of  $S$ , we can describe two functions that given relations from  $S$  to the set  $F(S)$  of all multiplicatively element. The set  $F(S)$  is formed all multiplicatively element of  $S$  given two function, that are  $\lambda$  and  $\phi$ . The  $\lambda$  function is a relation from  $a$  to  $a^\circ a$  and the  $\phi$  function is a relation from  $a$  to  $aa^\circ$ , and those functions satisfy  $\lambda^2 = \lambda$  and  $\phi^2 = \phi$ . Those functions are idempotent.

**Lemma 3.3.** *For a in semiring  $S$ , we have the following properties*

- (i)  $a\lambda(a) = aa^\circ a = a = \phi(a)a$
- (ii)  $\lambda(a)a^\circ = a^\circ aa^\circ = a^\circ = a^\circ \phi(a)$

*Proof.* Let  $S$  is semiring and  $a \in S$ . We have the following results:

- (i)  $a\lambda(a) = aa^\circ a = aa^*aa^*a = aa^*a = a$  and  $a\lambda(a) = aa^\circ a = \phi(a)a$ .
- (ii)  $\lambda(a)a^\circ = (a^\circ a)a^\circ = a^*aa^*aa^*aa^* = a^\circ aa^\circ = a^\circ$  and  $\lambda(a)a^\circ = a^\circ aa^\circ = a^\circ \phi(a)$ .

□

**Theorem 3.4.** *Let  $S$  is semiring and  $a, b$  are multiplicatively-regular elements in  $S$ . The form  $ax = b$  has a solution if and only if  $b$  satisfies  $\phi(a)b = b$ .*

*Proof.* ( $\Rightarrow$ ) Let  $ax = b$  has a solution, that is  $a^\circ b = x$ . We have,  $\phi(a)b = \phi(a)ax = aa^\circ ax = ax = b$ . Therefore, we have  $b$  that satisfies  $\phi(a)b = b$ .

( $\Leftarrow$ ) Let  $b$  satisfies  $\phi(a)b = b$ . We have,  $ax = a(a^\circ b) = \phi(a)b = b$ . It means that  $ax = b$  has a solution, that is  $a^\circ b = x$  □

**Definition 3.5.** *Let  $a$  is element in semiring  $S$ . The form  $a^r$  is called right complement if that element satisfies  $aa^r = 0$  and  $a + a^r = 1$ . Also, the form  $a^l$  is called left complement if that element satisfies  $a^l a = 0$  and  $a^l + a = 1$ .*

From that definition, we have the following property.

**Lemma 3.6.** *Let  $S$  is semiring. If  $a \in S$  has both right complement  $a^r$  and left complement  $a^l$  then  $a^r = a^l$ .*

*Proof.* We have

$$\begin{aligned} a^l &= a^l 1 = a^l (a + a^r) = a^l a + a^l a^r = a^l a^r \\ &= 0 + a^l a^r = aa^r + a^l a^r = (a + a^l) a^r = a^r. \end{aligned}$$

□

**Theorem 3.7.** *If  $a$  is multiplicatively-regular element of semiring  $S$  that satisfies  $\lambda(a)\lambda(a)^r = 0$ ,  $\lambda(a) + \lambda(a)^r = 1$ , and  $b \in S$  satisfies  $\phi(a)b = b$ , then solution of  $ax = b$  is  $\phi_b(y) = a^\circ b + \lambda(a)^r y$ .*

*Proof.* We have

$$\begin{aligned} a\phi_b(y) &= a(a^\circ b + \lambda(a)^r y) \\ &= aa^\circ b + a\lambda(a)^r y \\ &= \phi(a)b + a\lambda(a)\lambda(a)^r y \\ &= \phi(a)b, \text{ with } \lambda(a)\lambda(a)^r = 0 \\ &= b \end{aligned}$$

□

### 3.2. Existence of Solution of The Equations System with The Multiplicatively-Reguler Matrix

To develop the characteristics of the matrix such that the equation linear system  $AX = B$  has solution, it is carried through explores the matrix by referring to the properties of the multiplicatively-reguler elements and complement elements in semiring  $S$ . Let  $S$  is semiring and  $M_{n \times n}(S)$  is matrix over semiring  $S$ . The multiplicatively-reguler matrix  $A$  is  $A^*$  which satisfies  $AA^*A = A$ . If there exist a multiplicatively-reguler matrix then  $A^\circ = A^*AA^*$ , so we have  $AA^\circ A = AA^*AA^*A = AA^*A = A$ .

**Theorem 3.8.** *Let  $A$  and  $B$  are matrices over semiring  $S$  and  $A$  is multiplicatively-reguler matrix that satisfies  $AA^r = 0$  and  $AA^\circ B = B$ , then the linear equation  $AX = B$  has solution  $X = A^\circ B + A^r A$ .*

*Proof.* Let  $S$  is semiring. We have  $A$  is multiplicatively-reguler matrix that satisfies  $AA^r = 0$  and  $AA^\circ B = B$ . Furthermore, we have the following form.

$$\begin{aligned} AX &= A(A^\circ B + A^r A) \\ &= AA^\circ B + AA^r A \\ &= B + 0 = B \end{aligned}$$

Therefore,  $X = A^\circ B + A^r A$  is solution of  $AX = B$  with  $A$  and  $B$  are matrices over semiring  $S$ .  $\square$

## 4. Conclusion

Through explores the matrix by referring to the properties of the multiplicatively-reguler elements and complement elements in semiring  $S$ , we develop the characteristics of the matrix such that the equation linear system  $AX = B$  has solution. Let  $S$  is semiring and  $M_{n \times n}(S)$  is matrix over semiring  $S$ . The multiplicatively-reguler matrix  $A$  is  $A^*$  which satisfies  $AA^*A = A$ . If there exist a multiplicatively-reguler matrix then  $A^\circ = A^*AA^*$ , so we have  $AA^\circ A = AA^*AA^*A = AA^*A = A$ . From above characteristic, we get conclusion, that is, for  $A$  and  $B$  are matrices over semiring  $S$  and  $A$  is multiplicatively-reguler matrix that satisfies  $AA^r = 0$  and  $AA^\circ B = B$ , then the linear equation  $AX = B$  has solution  $X = A^\circ B + A^r A$ .

## Acknowledgements

This research was sponsored by the "Direktorat Pengelolaan Kekayaan Intelektual Direktorat Jenderal Penguatan Riset dan Pengembangan", by the "Indonesia program" on Ristekdikti. The scientific responsibility rests with its authors.

## References

- [1] Anton, H., 1987. *Elementary Linear Algebra*. John Wiley and Sons New York, Inc., USA.
- [2] Ariyanti, G., Suparwanto, A., and Surodjo, B., 2015. Necessary and Sufficient Conditions for The Solution of The Linear Balanced Systems in The Symmetrized Max Plus Algebra. *Far East J. Math. Sci.* (FJMS) 97(2), 253-266.
- [3] Brewer, J.W., Bunce, J.W., and Van Vleck, F.S. 1986. *Linear Systems over Commutative Rings*. New York : Marcel Dekker.
- [4] Brown, W. C. 1993. *Matrices over Commutative Rings*. New York : Marcel Dekker, Inc.
- [5] Narendran, P. 1996. Solving Linear Equations Over Polynomial Semirings. p. 466-472 in : "11 th Anual IEEE Symposium on Logic in Computer Science", IEEE Computer Society Press, Los Alamitos, CA.
- [6] Poplin, Philip L. 2000. *The Semiring of Multisets*. A thesis submitted to the Graduate Faculty of North Caroline State University.